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Dusty-gas flow in a laminar boundary layer over a blunt body

By E. S. ASMOLOV

Zhukovsky Central Aero-Hydrodynamics Institute, Zhukovsky, Moscow region, 140160, Russia

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Dusty-gas flow in a laminar boundary layer over a body with a curved surface is considered. In addition to Stokes drag, particles experience a centrifugal force and lift which is due to fluid shear. The body size L is taken to be much greater than the relaxation length of the particle velocity due to the action of Stokes drag, λ_{St}, and is of the same order as or less than the relaxation length due to the action of lift, λ_{Sa}. Using an asymptotic approach, momentum equations for the particle phase are reduced to an algebraic equation accounting for the variation of lift coefficient with the shear and the slip velocity. Particle velocity and density are computed for the axisymmetric boundary layer in the neighbourhood of the front stagnation point of a blunt body of size much less than λ_{Sa}. It is shown that downstream of some point on the wall (the separation point) particle normal velocity becomes non-zero. As a result particle streamlines turn away from the wall, and a particle-free zone arises. The cause of separation is the lift effect; the centrifugal force cannot make the particle flow separate. This conclusion is extended to the case when $L \sim \lambda_{Sa}$. The position of separation for the flow past a sphere is evaluated as a function of the ratio of its radius r' and relaxation length. Dust flow ceases to separate when this value is greater than a critical value $r'_c/\lambda_{Sa} \approx 29.2$.

1. Introduction

In most previous publications the problem of a dusty-gas boundary layer (Marble 1962; Saffman 1962; Singleton 1965; Osiptsov 1980; Wang & Glass 1988) was studied on the assumption that particles are under the action of a drag force only. However they also experience a lift which is due to fluid shear. This force is small compared with the longitudinal Stokes drag, but it can be of the order of the normal drag, and as a result the particle normal velocity can differ significantly from that of the fluid.

The effect of lift (it is also called 'Saffman's force' in Russian papers) on particle motion was considered for dusty-gas flow in laminar boundary layer over a flat plate by Otterman & Lee (1970) and Osiptsov (1988). The lift force coefficient was anticipated to be equal to the limiting value 6.46 obtained by Saffman (1965), but its dependence on the relation between the two particle Reynolds numbers based on slip velocity and on fluid shear (Asmolov 1990; McLaughlin 1991) was not taken into account. Dusty-gas flow in a boundary layer is assumed to have a single longitudinal length scale which is equal to the particle velocity relaxation length under the action of Stokes drag, λ_{St}. The lift force results in particle migration toward the plate at a distance from the leading edge of the order of λ_{St}, while far downstream its influence is negligible.

It was shown for the flow over a flat plate that the asymptotic approach requires the introduction of two distinguishing relaxation length scales, λ_{St} and λ_{Sa}, that correspond to the relaxation of the streamwise and normal velocity of particles, with λ_{Sa} being much greater than λ_{St} (Asmolov 1992). The leading-order streamwise velocity of particles at a distance from the leading edge of the order of λ_{Sa} is equal to that of the gas, i.e. particle flow in the streamwise direction is frozen in the fluid. An asymptotically small slip velocity is sufficient to make the lift comparable with the normal drag. Then the leading-order normal velocity of the particles differs from that of the gas. Momentum equations for the dispersed phase can be transformed to a single algebraic equation accounting for the variation of the lift-force coefficient c with the shear and the slip velocity. Dusty-gas flow in a boundary layer over a wedge was also considered in the framework of the asymptotic method (Asmolov 1993*b*). In this case the particle phase experiences a positive lift force. As a result its zeroth streamline turns away from the wall, and a particle-free zone adjacent to the wall arises.

The effect of lift force has been investigated previously only for flows over plane surfaces. In examining particle motion over a blunt body, when the boundary-layer equations are written in a coordinate system coupled with the body surface, one more force in addition to drag and lift should be accounted for – the centrifugal force. This more complicated flow configuration can also be considered on the basis of the method outlined when the characteristic size of streamlined body L is much greater than λ_{St}.

The outer inviscid problem for the case $\sigma \ll 1$, where $\sigma = \lambda_{St}/L$ is the Stokes number, was studied using an asymptotic approach by Michael (1968). For the incompressible flow of dusty gas past a sphere it was concluded that a dust-free layer adjacent to the sphere exists because of the action of the centrifugal force. However, as shown in the present paper this force can result in the particle density near the wall being asymptotically small but non-zero, i.e. it cannot force the dust flow to separate from the wall. The real cause of separation is the lift.

Similar to the Stokes number, a new dimensionless parameter can be introduced: $\theta = \lambda_{Sa}/L$. The effect of lift on particle motion in a boundary layer is significant when θ is of order of unity or greater. The incompressible axisymmetric flow in the neighbourhood of the front stagnation point of a blunt body for $\theta \gg 1$ is considered in more detail in §§4–6. In this case all normal forces acting on the particle are of the same order. This makes it possible to estimate the parts played by the different forces in particle motion and separation.

For simplicity the mass concentration of the dispersed phase is assumed to be small. Then the changes in the gas flow due to the particles can be neglected, and the problem can be solved using the momentum equations for the particle phase only. These latter are reduced to an algebraic equation. The particle normal velocity is evaluated from the numerical solution of this equation. Upstream of some point on the wall x_s (the separation point) it equals zero close to the body surface while downstream the normal velocity becomes non-zero. For this reason particles moving along the wall turn away from it at $x > x_s$, and separation of the dust flow occurs. Particle streamlines and the density distribution are calculated by integrating the equations of particle motion. The density in the vicinity of the boundary of the particle-free zone is several times greater than the free-stream value, and, hence, the particles are clustered near this boundary.

An asymptotic approach is extended to the case of arbitrary θ in §7. Separation of the particle phase again may arise owing to shear lift force only. As an example, the position of separation for boundary-layer flow of dusty gas past a sphere is evaluated as a function of the ratio of its radius and normal velocity relaxation length. Dust flow ceases to separate when this value is greater than the critical value $r'_c/\lambda_{Sa} \approx 29, 2$.

2. Lift force

Bretherton (1962) showed that at small Reynolds number no lateral force can be deduced on the basis of the creeping-flow equations whatever the undisturbed velocity profile. Small inertia effects must be taken into account to calculate this force. When the three Reynolds numbers based on fluid–particle translational velocity difference $(u'-u'_p)$, fluid shear $\partial u'/\partial y'$ and particle angular velocity Ω satisfy the following respective relations:

$$R_u = \frac{a\,|u'-u'_p|}{\nu} \ll 1, \quad R_k = \frac{a^2}{\nu}\frac{\partial u'}{\partial y'} \ll 1, \quad R_\Omega = \frac{a^2\Omega}{\nu} \ll 1, \quad R_\Omega \sim R_k, \quad R_u \sim R_k^{1/2},$$

it was found using the method of matched asymptotic expansions that the lift force is (Saffman 1965)

$$F_{Sa} = c_F(\alpha, Y)\, R_k^{1/2}\, \mu a (u'-u'_p),$$

where

$$\alpha = \mp R_u\, R_k^{-1/2}. \tag{2.1}$$

Here ν and μ are kinematic and dynamic viscosities of the gas, a is the particle radius, and Y is the distance between the particle and the wall scaled on the length of the outer region $l_e = aR_k^{-1/2}$. Parameter α characterizes the ratio of the two parts of the inertial term in the momentum equation corresponding to uniform (Oseen) and shear (Saffman) undisturbed flow. Its sign is taken to be the same as that of slip velocity $u'-u'_p$. Equation (2.1) can be rewritten in terms of the dimensional slip velocity and the fluid velocity gradient as

$$\alpha = (u'-u'_p)\left(\nu \frac{\partial u'}{\partial y'}\right)^{-1/2}.$$

Saffman (1965) found a limiting value of the lift force coefficient of $c_F = 6.46$ in unbounded shear flow for the case $\alpha \to 0$. For arbitrary α in unbounded flow the coefficient

$$c(\alpha) \equiv c_F(\alpha, Y \to \infty) \tag{2.2}$$

was calculated independently by Asmolov (1990) and McLaughlin (1991). For arbitrary distance Y lift was evaluated by Asmolov (1990) and later by McLaughlin (1993).

When particle motion in a laminar boundary layer is considered, the length scale of outer region l_e is estimated as

$$l_e \sim \left(\frac{1}{\nu}\frac{\partial u'}{\partial y'}\right)^{-1/2} \sim L\, Re_L^{-3/4}, \tag{2.3}$$

where

$$Re_L = \frac{VL}{\nu} \gg 1.$$

Here V is a characteristic gas velocity. It can be seen from (2.3) that for any longitudinal length scale of the boundary layer L its thickness $\delta = L\, Re_L^{-1/2}$ is much greater than l_e. From this two important conclusions follow. First, the wall effect is significant only in a thin sublayer with the thickness l_e. Secondly, the undisturbed flow past a single particle in the major portion of the boundary layer excluding this sublayer can be treated as unbounded simplc shear flow, and the lift coefficient (2.2) can be taken to describe particle motion.

The $c(\alpha)$ computed by Asmolov (1990) and McLaughlin (1991) coincide within the accuracy of the calculations in the interval $0 \leqslant |\alpha| \leqslant 3$. The lift coefficient decreases

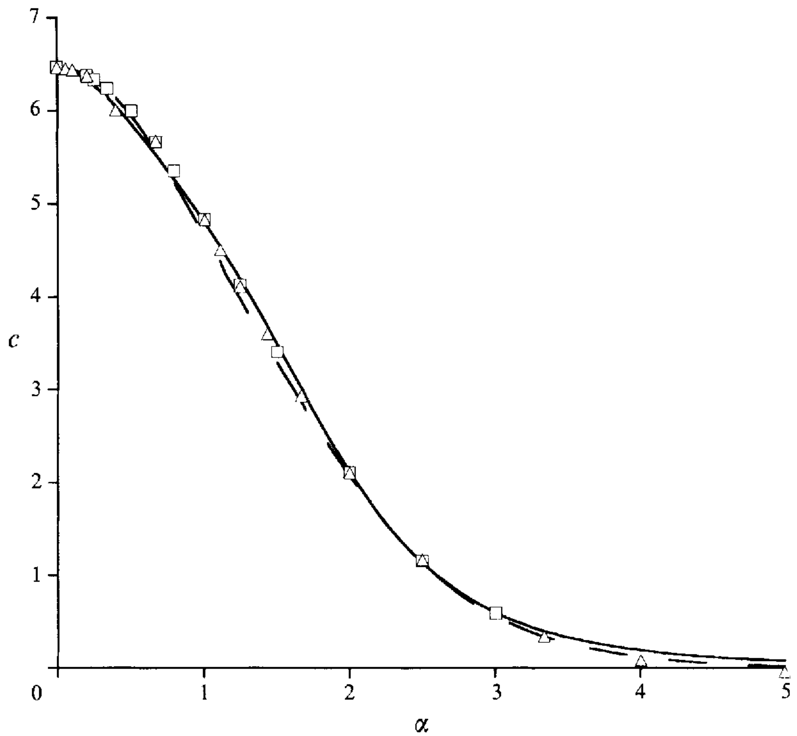

FIGURE 1. Comparison between the lift force coefficients computed by Asmolov (1990) (□), McLaughlin (1991) (△) and interpolations by Asmolov (1992) (solid line), and Mei (1992) (dashed line).

monotonically from the limiting value $c(0) = 6.46$ to $c(3) = 0.58$. It changes sign for $|\alpha| > 4.5$ and rapidly approaches zero, i.e. $c(\alpha) \sim \alpha^{-5} \log \alpha$ as $\alpha \to \infty$ (McLaughlin 1991). Two interpolations of $c(\alpha)$ have been proposed: by Asmolov (1992)

$$c(\alpha) = 6.46\,(1 + 0.581\,\alpha^2 - 0.439\,|\alpha|^3 + 0.203\,\alpha^4)^{-1}, \quad 0 \leqslant |\alpha| \leqslant 3, \tag{2.4}$$

and by Mei (1992)

$$c(\alpha) = 1.936\{1 + \tanh[2.5(0.19 - \log_{10}|\alpha|)]\} \times \{0.667 + \tanh[6(|\alpha|^{-1} - 0,32)]\}, \quad 0.05 \leqslant |\alpha| \leqslant 10. \tag{2.5}$$

The lift coefficients derived from (2.4) and (2.5) are shown in figure 1. Both interpolations agree closely with numerical values in the interval where c is of order of unity. Equation (2.4) deviates from the calculated coefficient at $|\alpha| > 3$. Nevertheless we use this expression below for arbitrary α since the magnitude of lift at $|\alpha| > 3$ is very small, and its effect on particle motion can be neglected.

3. Length scales and governing equations

Consider dispersed-phase flow which consists of particles of the same radius a and has a uniform upstream velocity V and particle mass density

$$\rho_{p\infty} = \tfrac{4}{3}\pi n_{p\infty} a^3 \rho_s,$$

which is much less than that of the gas, ρ. Here n_p is the number density of the particles, ρ_s is the mass density of their material.

A critical question for the problem being discussed is the choice of a characteristic length of the flow. There are the time and length (Marble 1962; Saffman 1962)

$$\tau_{St} = \tfrac{2}{9}\rho_s a^2/\mu, \quad \lambda_{St} = \tau_{St} V \tag{3.1}$$

of particle velocity relaxation under the action of Stokes force. Flow in a boundary layer over a flat plate has no length scale of the variation of fluid velocity in the outer inviscid region, and for this reason λ_{St} is customarily taken as the characteristic length for dusty-gas flow. The Reynolds number based on this length is assumed to be the single asymptotic parameter

$$Re_{St} = \frac{V\lambda_{St}}{\nu} \gg 1. \tag{3.2}$$

Taking account of (3.1), equation (3.2) can be rewritten in terms of the mass densities ratio and the particle Reynolds number defined as

$$\beta = \frac{\rho}{\rho_s}, \quad R_V = \frac{Va}{\nu}$$

in the following form:

$$Re_{St} = \tfrac{2}{9} R_V^2 \beta^{-1} \gg 1. \tag{3.3}$$

It can be easily seen that (3.3) implies the existence at least one more asymptotic variable, namely

$$\beta \ll 1 \quad \text{or (and)} \quad R_V^{-1} \ll 1. \tag{3.4}$$

Below, both parameters β and R_V^{-1} are taken to be asymptotically small. Equation (3.4) means that to use an asymptotic approach properly it is necessary to estimate the values of the different forces and migration velocity more carefully. Assuming the lift coefficient to be of the order of unity one can easily obtain

$$F_{St}^y(\lambda_{St}) \sim \mu a(v' - v_p') \sim \mu a V Re_{St}^{-1/2} \sim \mu a V R_V^{-1} \beta^{1/2},$$
$$F_{Sa}(\lambda_{St}) \sim \mu a(u' - u_p') R_k^{1/2}(\lambda_{St}) \sim \mu a V R_V Re_{St}^{-1/4} \sim \mu a V R_V^{1/2} \beta^{1/4}.$$

Therefore, if either of the inequalities (3.4) are satisfied, normal drag is less than lift at distances from the leading edge of the order of λ_{St}. From this it follows that the two normal forces are comparable far downstream where the streamwise slip velocity is small. A new length scale λ_{Sa} different from λ_{St} should be introduced. For particle motion in a boundary layer over a flat plate $\lambda_{Sa,p}$ was defined in Asmolov (1992) as

$$\lambda_{Sa,p} = 3.56 \times 10^{-3} (V/\mu)^3 a^4 \rho^{5/3} \rho_s^{4/3}. \tag{3.5}$$

$\lambda_{Sa,p}$ and the corresponding Reynolds number $Re_{Sa,p}$, which follows from (3.5), are much greater than λ_{St} and Re_{St}, since

$$\lambda_{Sa,p} \sim \beta^{-1/3} R_V^2 \lambda_{St} \gg \lambda_{St}, \quad Re_{Sa,p} = \frac{V\lambda_{Sa,p}}{\nu} \sim \beta^{-4/3} R_V^4 \gg Re_{St}. \tag{3.6}$$

At distances of the order of $\lambda_{Sa,p}$ the two normal forces are of the same order so that

$$F_{St}^y(\lambda_{Sa,p}) \sim \mu a v' \sim \mu a V Re_{Sa,p}^{-1/2} \sim \mu a V R_V^{-2} \beta^{2/3},$$
$$F_{Sa}(\lambda_{Sa,p}) \sim \mu a(u' - u_p') R_k^{1/2}(\lambda_{Sa,p}) \sim \mu a V R_V^{-2} \beta^{2/3},$$

since

$$u' - u_p' \sim V \frac{\lambda_{St}}{\lambda_{Sa,p}} \sim V \beta^{1/3} R_V^{-2} \ll V,$$

and the migration velocity is of the order of the normal velocity of the gas. Length scales λ_{St} and $\lambda_{Sa,p}$ can be interpreted respectively as the relaxation lengths of the streamwise u_p and normal v_p velocities. Equations (3.6) provide the basis for the asymptotic description of the particle motion. Momentum equations for the disperse phase within the framework of this asymptotic approach can be reduced to a single algebraic equation that accounts for the variation of the lift force coefficient c with the shear and slip velocity.

Describing particle motion in a boundary layer over a body with a curved surface is a more complicated problem since one more length scale (a characteristic size of the body L) and one more force (centrifugal force) must be taken into account in addition to above-listed lengths and forces. Nevertheless, an asymptotic approach can be successfully used to solve such a problem when the characteristic size of the streamlined body L is much greater than λ_{St} or, equivalently, the Stokes number $\sigma = \lambda_{St}/L \ll 1$.

We use a common coordinate system which is coupled with the body surface. The x-axis is directed along the surface while the y-axis is normal to it. If the particle motion is considered in this coordinate system a centrifugal force arises. Then the dimensionless momentum equations for the particle phase can be written as

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{u-u_p}{\sigma}, \tag{3.7a}$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{v-v_p}{\sigma} + Re_L^{1/2} \frac{u_p^2}{r} + \frac{c(\alpha)(u-u_p)}{\epsilon} \left(\frac{\partial u}{\partial y}\right)^{1/2}, \tag{3.7b}$$

where $$Re_L \gg 1, \quad \sigma = \tfrac{2}{9} Re_L^{-1} R_V^2 \beta^{-1}, \quad \epsilon = \tfrac{4}{3}\pi Re_L^{-5/4} R_V \beta^{-1},$$

$$\alpha = \left(\frac{\partial u}{\partial y}\right)^{-1/2} (u-u_p)\, Re_L^{1/4}.$$

Here the dimensionless variables are introduced in a common way, namely

$$x = \frac{x'}{L}, \quad y = \frac{Re_L^{1/2} y'}{L}, \quad r = \frac{r'}{L}, \quad u = \frac{u'}{V}, \quad v = \frac{Re_L^{1/2} v'}{V}, \quad u_p = \frac{u'_p}{V}, \quad v_p = \frac{Re_L^{1/2} v'_p}{V},$$

where L is a characteristic size of the body, and $r'(x')$ is its local radius of curvature. The velocity field of the gas can be described as unperturbed by the particle motion since the particle mass fraction is small compared with unity.

In most previous investigations primary attention was given to the case when the Stokes number $\sigma \sim 1$, i.e. the size of a streamlined body is comparable with the relaxation length λ_{St}. It was thought that the particle velocity differs from that of the gas in this case only. For bodies of larger size the slip velocity is assumed to be small. Indeed we have from equation (3.7a) for streamwise component

$$u - u_p \sim \sigma u \ll 1 \quad \text{when} \quad \sigma = \frac{2a^2}{9} \frac{\rho}{\nu} \frac{V}{\rho_s L} \ll 1. \tag{3.8}$$

The last term on the right-hand side of (3.7b) that corresponds to the lift force and is proportional to slip velocity can be estimated in view of (3.8) as

$$\sigma\epsilon^{-1} \sim Re_L^{1/4} R_V \gg 1 \quad \text{when} \quad Re_L \gg 1 \quad \text{and} \quad R_V \gg 1. \tag{3.9}$$

Therefore this term is large even for a small slip velocity. Assuming that $v - v_p \sim 1$ the term corresponding to normal drag can be estimated as

$$\sigma^{-1} \sim Re_L R_V^{-2} \beta. \tag{3.10}$$

Two terms are comparable when

$$\sigma\epsilon^{-1} \sim \sigma^{-1}.$$

The last equation is valid when $Re_L \sim \beta^{-4/3} R_V^4$. We introduce the relaxation length of the normal velocity λ_{Sa} as the length L that makes

$$\sigma\epsilon^{-1} = \sigma^{-1}, \tag{3.11a}$$

or, equivalently,

$$\tfrac{1}{6}\pi^{-1} Re_L^{1/4} R_V = \tfrac{9}{2} Re_L R_V^{-2} \beta. \tag{3.11b}$$

Denote Re_{Sa} as the Reynolds number Re_L that satisfies equation (3.11*b*). This gives

$$Re_{Sa} = \tfrac{1}{81}(\pi\beta)^{-4/3} R_V^4. \tag{3.12a}$$

Defining a length scale λ_{Sa} such that $Re_{Sa} = V\lambda_{Sa}/\nu$, one obtains from (3.12*a*)

$$\lambda_{Sa} = \tfrac{1}{81}\pi^{-4/3}(V/\mu)^3 a^4 \rho^{5/3} \rho_s^{4/3}. \tag{3.12b}$$

Thus in this case the relaxation length λ_{Sa} differs from that for the flow over flat plate, $\lambda_{Sa,\,p}$, only by a numerical coefficient and is again large compared with λ_{St}. It follows from (3.9), (3.10) that normal drag increases more rapidly with L than lift. Hence the lift effect on particle motion in the boundary layer is significant when L is of order of λ_{Sa} or less.

What forces will be dominant depends not only on the ratio of L and λ_{Sa} but also on the relation between asymptotic parameters R_V^{-1} and β. Among all the possible cases, we consider in more detail the case when normal drag, lift and centrifugal forces are of the same order. Another requirement is that normal slip velocity due to their action is of the order of v. At $x \sim 1$ we have $u \sim 1$, $v \sim 1$, and the three terms on the right-hand side equation (3.7*b*) are of the same order when

$$Re_L \beta R_V^{-2} \sim Re_L^{1/2} \sim Re_L^{1/4} R_V. \tag{3.13a}$$

In addition, to demonstrate the influence of the variation of the lift coefficient c on the characteristic features of particle motion we make the particle Reynolds numbers ratio α to be approximately unity. In the view of (3.9) one can obtain

$$\alpha \sim Re_L^{-3/4} \beta^{-1} R_V^2 \sim 1. \tag{3.13b}$$

One can easily conclude that three independent parameters, Re_L, R_V, β, are not sufficient to determine (3.13*a, b*). The above requirements can be satisfied by letting the streamwise velocity be asymptotically small so that $u \sim \tau \ll 1$. This is valid for the flow near the front stagnation point of a blunt body at the distance $x \sim \tau$ from the stagnation point. In this case the second and third terms on the right-hand side of (3.7*b*) and α should be re-estimated. The centrifugal force is quadratic in u, lift is proportional to $u(\partial u/\partial y)^{1/2}$ and, hence, to $\tau^{3/2}$. For α proportional to $u(\partial u/\partial y)^{-1/2}$ we have $\alpha \propto \tau^{1/2}$. Then (3.13*a, b*) are rewritten as

$$Re_L \beta R_V^{-2} \sim \tau^2 Re_L^{1/2} \sim \tau^{3/2} Re_L^{1/4} R_V, \tag{3.14a}$$

$$\alpha \sim \tau^{1/2} Re_L^{-3/4} \beta^{-1} R_V^2 \sim 1. \tag{3.14b}$$

Equations (3.14*a, b*) are satisfied if the main asymptotic parameters are related as follows:

$$\beta \sim Re_L^{-1/2}, \quad R_V \sim Re_L^{1/6}, \quad \tau \sim Re_L^{-1/6}. \tag{3.15}$$

Other asymptotic parameters are

$$\sigma \sim Re_L^{-1/6}, \quad \epsilon \sim Re_L^{-7/12}.$$

Equation (3.15) means that the characteristic size of a streamlined body is much less than λ_{Sa} since

$$\theta = \lambda_{Sa}/L \sim Re_L^{-1}\beta^{-4/3} R_V^4 \sim Re_L^{1/3} \gg 1,$$

where λ_{Sa} is given by (3.12*b*). Equation (3.15) defines only the order of τ, not its exact value. Hence it can be introduced without loss in generality as $\tau = \theta^{-1/2}$.
This last definition does not contradict (3.15).

The lift force can be evaluated provided that the particle Reynolds numbers R_u and R_k are small. These conditions are satisfied for the above relations between the main asymptotic parameters since

$$R_u \sim R_V \tau\sigma \sim Re_L^{-1/6} \ll 1, \quad R_k \sim R_V^2 \tau\, Re_L^{-1/2} \sim Re_L^{-1/3} \ll 1.$$

Thus the use of expressions for the lift given in §2 is justified in this case. It should also be noted that relations (3.15) are quite possible in practice. For example, an air flow containing water droplets of radius $a = 5\times 10^{-6}$ m with the characteristic velocity $V = 100$ m s^{-1} and size of a streamlined body $L = 0.2$ m at standard conditions corresponds to relaxation length $\lambda_{Sa} \approx 3$ m and the following values of the dimensionless parameters:

$$Re_L \approx 10^6, \quad \beta \approx 10^{-3}, \quad R_V \approx 30, \quad \tau \approx 0.2, \quad \theta \approx 15, \quad \sigma \approx 0.2, \quad \epsilon \approx 0.004.$$

The flow in the vicinity of the stagnation point of a body with size much less than λ_{Sa} is the most illustrative case. All the terms corresponding to the various forces in the algebraic equation for α derived in the next section are of the same order. This makes it possible to estimate the parts played by these forces in particle motion and separation. The asymptotic approach is extended in §7 to the general case when Re_L, R_V, β are related differently than in (3.15), and some terms can be neglected.

4. Particle motion near the stagnation point

The size of the body, L, can be taken without loss in generality to equal the radius of the curvature of body surface at the stagnation point $r'(0)$, so that $r(0) = 1$. Introducing new dimensionless variables

$$X = \tau^{-1}x, \quad U = \tau^{-1}u, \quad U_p = \tau^{-1}u_p,$$

and coefficients

$$\gamma = \sigma\, Re_L^{1/4}\tau^{1/2} = \tfrac{2}{3}\pi^{1/3}\, Re_L^{-1/2}\beta^{-2/3} R_V \sim 1,$$
$$\kappa = \sigma\, Re_L^{1/2}\tau^2\gamma = 12\,\pi^{5/3}\beta^{-1/3} R_V^{-1} \sim 1,$$

one can rewrite (3.7) as

$$U_p\frac{\partial U_p}{\partial X} + v_p\frac{\partial U_p}{\partial y} = \frac{U-U_p}{\sigma}, \tag{4.1a}$$

$$U_p\frac{\partial v_p}{\partial X} + v_p\frac{\partial v_p}{\partial y} = \frac{v-v_p}{\sigma} + -\frac{\kappa}{\gamma}\frac{U_p^2}{\sigma} + \frac{c(\alpha)(U-U_p)}{\sigma^2}\left(\frac{\partial U}{\partial y}\right)^{1/2}, \tag{4.1b}$$

$$\alpha = \gamma\left(\frac{\partial U}{\partial y}\right)^{-1/2}\frac{U-U_p}{\sigma}. \tag{4.2}$$

Solution of (4.1) is sought in the form

$$U_p = U_{p0} + \sigma U_{p1} + \ldots, \quad v_p = v_{p0} + \sigma v_{p1} + \ldots, \quad \alpha = \alpha_0 + \sigma\alpha_1 + \ldots.$$

The left-hand sides of (4.1*a, b*) are asymptotically small compared with the right-hand sides. Consequently, the zeroth order of both drag and net transverse force are to equal zero. From (4.1*a*) one can deduce for the streamwise particle velocity

$$U_{p0} = U. \tag{4.3}$$

Thus the dispersed phase in the streamwise direction is frozen into the fluid. Such a quasi-equilibrium motion of the particles does not imply, however, their leading-order normal velocity to be equal to that of the fluid. The term in (4.1) corresponding to lift is of order σ^{-2}, and for this reason even a small slip velocity,

$$U-U_p = -\sigma U_{p1}+O(\sigma^2),$$

makes it comparable with terms corresponding to normal drag and centrifugal force. Therefore knowing U_{p0} one cannot evaluate v_{p0}: it is necessary to account for the first-order streamwise velocity. Collecting the terms of power σ^0 in (4.1 *a*) and power σ^{-1} in (4.1 *b*) we obtain in view of (4.3)

$$U\frac{\partial U}{\partial X}+v_{p0}\frac{\partial U}{\partial y} = -U_{p1}, \tag{4.4a}$$

$$v-v_{p0}+\frac{\kappa}{\gamma}U^2-c(\alpha_0)\,U_{p1}\left(\frac{\partial U}{\partial y}\right)^{1/2} = 0. \tag{4.4b}$$

U_{p1} and v_{p0} can be expressed, taking account of (4.2) and (4.4) in terms of α_0 as

$$U_{p1} = -\frac{\alpha_0}{\gamma}\left(\frac{\partial U}{\partial y}\right)^{1/2}, \quad v_{p0} = v+\frac{\kappa}{\gamma}U^2+\frac{c(\alpha_0)\,\alpha_0}{\gamma}\left(\frac{\partial U}{\partial y}\right). \tag{4.5}$$

Substituting (4.5) into (4.4) one can reduce these equations to a single one:

$$c(\alpha_0)\,\alpha_0\left(\frac{\partial U}{\partial y}\right)^2 = \alpha_0\left(\frac{\partial U}{\partial y}\right)^{1/2}-\gamma\frac{\mathrm{D}U}{\mathrm{D}t}-\kappa U^2\frac{\partial U}{\partial y}, \tag{4.6}$$

where

$$\frac{\mathrm{D}U}{\mathrm{D}t} = U\frac{\partial U}{\partial X}+v\frac{\partial U}{\partial y}$$

is the streamwise acceleration evaluated along the fluid streamline. The gas velocity distribution near the stagnation point is (Schlichting 1968)

$$U = \tfrac{3}{2}X\phi'(\eta), \quad v = -\surd 3\,\phi(\eta).$$

Here $\eta = \surd 3y$, function $\phi(\eta)$ is the solution of the Faulkner–Scan equation by virtue of the known similarity between boundary-layer flows near the stagnation point and over a wedge, and $\phi'(\eta)$ denotes its derivative. Finally, omitting henceforth subscript 0, one can rewrite (4.6), (4.5) as

$$\chi(\alpha) = \alpha Q(X,\eta)+Z(X,\eta), \tag{4.7}$$

$$v_p = \surd 3\left[-\phi+\tfrac{3}{2}X\frac{\chi(\alpha)}{\gamma}\phi''\right]+\frac{9}{4}\frac{\kappa}{\gamma}X^2\phi'^2, \tag{4.8}$$

where

$$\chi = c(\alpha)\,\alpha, \quad Q = X^{-3/2}q(\eta), \quad Z = -\gamma X^{-1}s(\eta)-\kappa Xp(\eta),$$

$$q = (\tfrac{3}{2}\surd 3\phi'')^{-3/2}, \quad s = \frac{1}{3}\left[\left(\frac{\phi'}{\phi''}\right)^2-2\frac{\phi}{\phi''}\right], \quad p = \frac{\surd 3}{2}\frac{\phi'^2}{\phi''}.$$

All functions q, s, p depending on η are positive ones.

Thus the particle velocity field can be evaluated from the solution of the algebraic equation (4.7). Similar equations were obtained for dusty-gas flow over a flat plate and a wedge by Asmolov (1992, 1993 *b*). The distinction of (4.7), (4.8) is the terms proportional to κ, corresponding to the centrifugal force. The main features of this equation such as non-uniqueness of solution and peculiarities of distributions α and v_p near the wall are due to function $\chi(\alpha)$ on its left-hand side. For this reason they are just the same for the problem in question as for a flow over a wedge.

5. Uniqueness and matching condition

Because of the nonlinearity of equation (4.7) governing the quasi-equilibrium motion of the particles its solution in the general case is not unique. This conclusion is illustrated in figure 2, where the solid line represents the graph of the function $\chi(\alpha)$ on the left-hand side of (4.7). The right-hand side is the linear function $\alpha Q + Z$ of α with the coefficients Q, Z depending on the coordinates X, η and the parameters γ, κ. The dashed straight lines 1–3 in figure 2 correspond to $\gamma = 1$, $\kappa = 2$, $X = 0.5$ and various values of η. The roots of (4.7) are located at the intersection of the dashed and solid lines. As seen in figure 2 there can be one (point P for $\eta = 1.5$), two (points O, N for $\eta = 1.15$) or three (points M, L, K for $\eta = 0.4$) roots depending on X, η. The roots are denoted by α_A (points P, N, K), α_B (point O), α_C (point M), so that $\alpha_A > \alpha_B > \alpha_C$. One of the roots (in this case α_A) exists over the whole boundary layer while the two others do not exist for all values of X, η.

A study of the stability of quasi-equilibrium motion to small perturbations of particle velocity (Asmolov 1993*a*) shows that the solution is stable when

$$\frac{\mathrm{d}\chi}{\mathrm{d}\alpha} < \left(\frac{\partial U}{\partial y}\right)^{-3/2}, \tag{5.1a}$$

or, equivalently,

$$\mathrm{d}\chi/\mathrm{d}\alpha < Q. \tag{5.1b}$$

Thus the solution is stable when the velocity gradient is less than a critical value given by

$$\left(\frac{\partial U}{\partial y}\right)_c = \left(\frac{\mathrm{d}\chi}{\mathrm{d}\alpha}\right)^{-2/3}. \tag{5.2}$$

Equation (5.1*b*) means that the inclination of a solid line in figure 2 at the intersection must be less than that of dashed line. This is valid for the two extreme roots α_A and α_C. The opposite inequality occurs for the middle root α_B. Consequently α_B is an unstable root, and for this reason it is never established. The instability of this branch can be explained as follows. Let the particle normal velocity be slightly perturbed with respect to the magnitude v_B corresponding to α_B. This results in the increase of not only normal drag, but, as follows from (4.4), also of the streamwise slip velocity and lift. If $\mathrm{d}\chi/\mathrm{d}\alpha > Q$ the increase of the lift due to the small perturbation is greater than that of the normal drag, and deviation from v_B increases further till the stable branch is established.

Another conclusion following from the stability analysis is that the solution varies continuously and remains on the same stable branch during the particle motion until its trajectory leaves the region of existence of this branch. Which stable branch (α_A or α_C) describes the real motion of the particle depends on the initial value of its velocity. This latter must be evaluated using the matching condition with the outer inviscid region. The Stokes force is dominant in this region. This is easily deduced from (4.7). In the limit $\eta \to \infty$ we have $\phi' \to 0$, $\phi'' \to 0$. Hence the term corresponding to lift, which is the cause of the non-uniqueness, is small compared with the right-hand side of (4.7). Thus the solution of (4.7) is unique and is given by

$$\alpha = \alpha_A \to \gamma X^{1/2} s q^{-1} \quad \text{as} \quad \eta \to \infty. \tag{5.3}$$

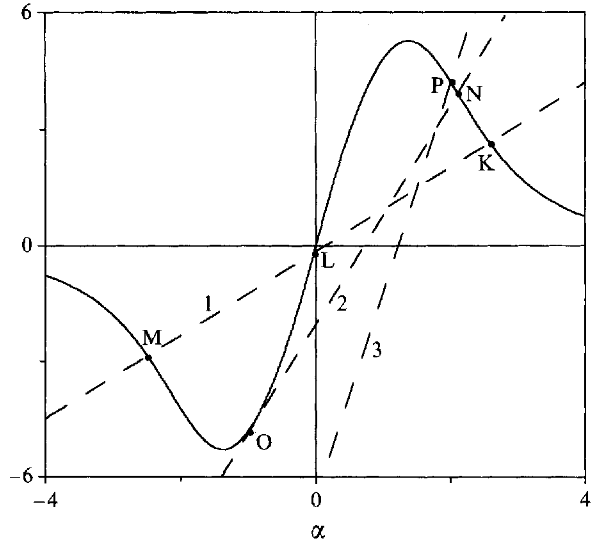

FIGURE 2. Graphic solution of equation (4.7). Solid line is its left-hand side, and the dashed ones are the right-hand side for $\gamma = 1$, $\kappa = 2$, $X = 0, 5$ and various η: 0.4; 1.15; 1.5, lines 1–3 respectively.

The same equation can be derived for particles in the immediate vicinity of the stagnation point when $X \to 0$. In this case (5.3) is valid since right-hand side of (4.7) is again large in comparison with the left-hand side in the limit $X \to 0$. Thus the motion of particles both on upper and leading edges of boundary layer is described by the positive branch α_A. This root exists for all X, η since the function $Z(X, \eta)$ is negative. From this it follows that particles continue to move in accordance with the same positive branch of the quasi-equilibrium solution, and solution α_A is valid everywhere in the boundary layer.

For both of the above limiting cases we can obtain for the normal particle velocity, in view of (4.8) and (5.3),

$$v_p \to v \quad \text{when} \quad X \sim 1, \quad \eta \to \infty \quad \text{or} \quad X \to 0, \quad \eta \sim 1.$$

Dust flow is frozen in the fluid in both regions, and for this reason the particle density equals its free-stream value.

In the far downstream region the lift effect is again small, and the centrifugal force is dominant since in the limit $X \to \infty$ the term corresponding to this force in (4.7) tends to infinity, being linearly proportional to X, while $\chi(\alpha)$ is finite for any α. Taking account of (4.8) one can obtain

$$\alpha \to \kappa X^{5/2} p q^{-1}$$

and

$$v_p \to \frac{9}{4}\frac{\kappa}{\gamma} X^2 \phi'^2 \quad \text{as} \quad X \to \infty, \quad \eta \sim 1.$$

Therefore the lift force is small for all limiting cases, and its influence is significant only when $X \sim 1$, $\eta \sim 1$.

It should be noted that the matching problem can be easily solved in the framework of this asymptotic method for finite particle content as well. Leading-order velocities of the two phases in the outer region are equal and the particle density field is homogeneous (Michael 1968). The matching problem is more complicated when the size of a body is assumed to be of order λ_{St}. In this case the slip velocity in outer region is finite. To determine the velocity and density of the dispersed phase on the upper edge of boundary layer it is necessary to solve the entire outer problem. The sole exception is a dusty-gas flow over a flat plate which has homogeneous velocity and particle density fields in the outer region.

6. Separation of particle flow

Close to the wall both the velocity and acceleration of the carrier gas tend to zero. One would expect α and v_p to show similar behaviour in η. The solution of (4.7) given by

$$\alpha(X,0) = 0 \tag{6.1}$$

exists for all X. It corresponds to non-slip flow. However, the zeroth solution is not always stable yet. The necessary condition (5.1) should be met for (6.1) to be a stable branch. We define X_s as the coordinate of the point where the fluid shear reaches the critical value (5.2) calculated for the zeroth solution. As $\mathrm{d}\chi(0)/\mathrm{d}\alpha = c(0)$ one can obtain

$$\left(\frac{\partial U}{\partial y}\right)_{c0} = c(0)^{-2/3},$$

and

$$X_s = \frac{2}{3}\frac{c(0)^{-2/3}}{\sqrt{3\phi''(0)}} \approx 0.12. \tag{6.2}$$

Then (5.1) is equivalent to $X < X_s$. Upstream of X_s fluid shear is less than a critical value, and (6.1) is stable. In this region there exists only the zeroth root of (4.7). For $X > X_s$ this root is not unique and corresponds to the unstable branch α_B which can be never established. In this case particle motion is governed by the non-zero positive branch α_A. Therefore because of the lift effect the streamwise slip velocity and, hence, particle migration velocity can be non-zero, even though the velocity of the carrier gas is zero.

These features of the distribution $\alpha_A(X,\eta)$ are illustrated in figure 3(*a*). The η-dependences of α evaluated numerically using Newton's method for $\gamma = 1$, $\kappa = 2$ and various values of X are presented by solid lines. For comparison, dependences $\alpha(\eta)$ calculated with neglect of the lift (the term on the left-hand side of (4.7) set to equal zero) are plotted by dashed lines. These dependences, in contrast to $\alpha_A(X,\eta)$, are quadratic in η as $\eta \to 0$ for all X.

The particle normal velocity field shows a similar behaviour near the wall. It follows from (6.1) and (4.8) that for $X < X_s$ we have $v_p \to 0$ as $\eta \to 0$, while for $X > X_s$, $v_p(X,0)$ is non-zero. The η-dependences of v_p calculated from (4.8) are presented by solid lines in figure 3(*b*) for the same values of γ, κ, X as in figure 3(*a*). Dashed lines represent the dependences of the particle normal velocity calculated with neglect of the lift (the second term in square brackets in (4.8) set to equal zero). It is seen that the lift effect is significant at $X > X_s$ and near the wall.

Particles moving along the wall at $X < X_s$ must move away from it at $X > X_s$ since $v_p(X,0)$ becomes non-zero. This phenomenon can be treated as the separation of dust flow near the stagnation point of the carrier-gas flow. To illustrate it some particle streamlines for $\gamma = 2$, $\kappa = 1$ are presented in figure 4. The zero streamline plotted by the heavy line is the boundary of separation, and a particle-free zone arises to its right. It is seen that other streamlines come close to this boundary, and particles are clustered

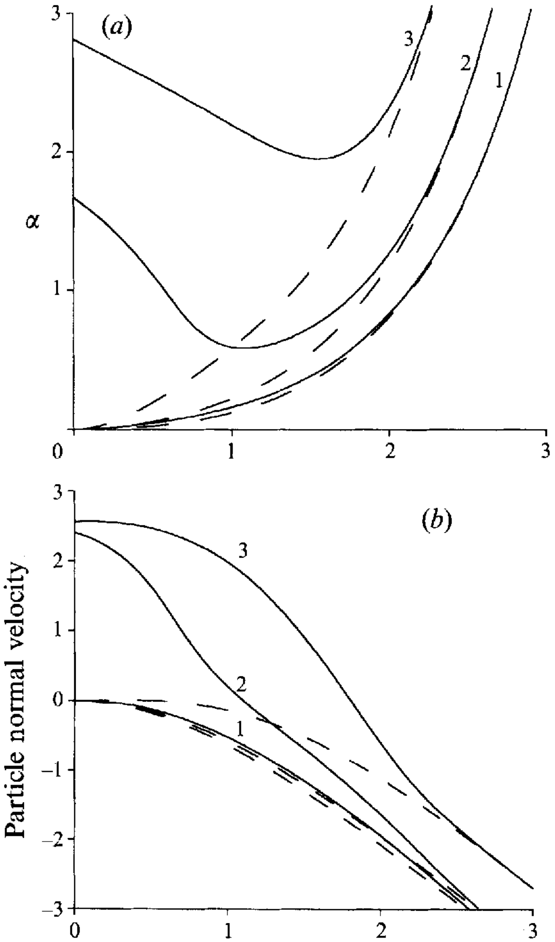

FIGURE 3. (*a*) Particle Reynolds number ratio α and (*b*) particle normal velocity at $\gamma = 1$, $\kappa = 2$ and various distances from the stagnation point $X = 0.1$; 0.2; 0.5 (corresponding to solid curves 1–3). Dashed lines are the same values calculated neglecting the lift.

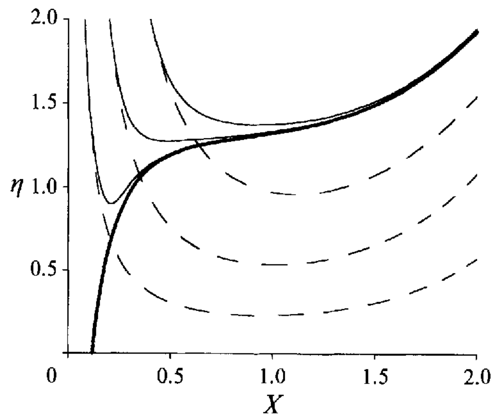

FIGURE 4. Particle streamlines (solid lines) and boundary of the particle-free zone (heavy line) at $\gamma = 2$, $\kappa = 1$. Dashed lines are the streamlines calculated neglecting the lift.

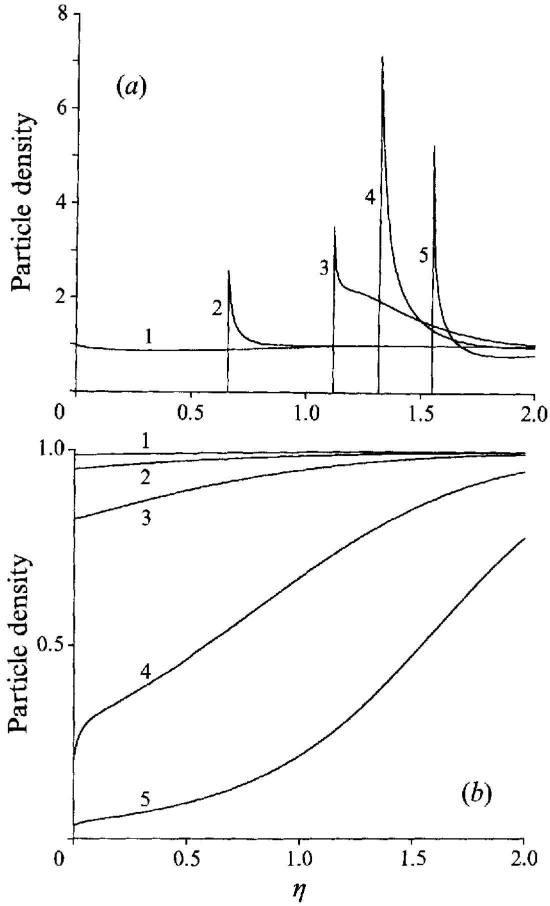

FIGURE 5. Particle density scaled by its free-stream value at $\gamma = 2$, $\kappa = 1$ and various distances from the stagnation point $X = 0.1$; 0.2; 0.5; 1; 1.6 (corresponding to curves 1–5) calculated (*a*) taking account of lift and (*b*) with its neglect.

near it. It will be recalled that in our analysis there are no particles very close to the wall in a thin sublayer with thickness $l_e \sim L\,Re_L^{-3/4}$. In this sublayer the wall effect on the lift force should be taken into account. Particle motion in this region requires special consideration.

The separation of dust flow and clustering of particles near the boundary of separation can also be followed from the calculation of the particle density distribution. To evaluate $\rho_p'(X, \eta)$ two nearby trajectories are plotted. The continuity of the particle phase means that for axisymmetric flow the product of the spacing between trajectories, tangent velocity, distance from axis of symmetry and particle density is invariant. The initial value of this invariant is determined on the upper edge of boundary layer (matching condition) where the particle velocity is equal to that of the fluid and the density equals its free-stream value $\rho_{p\infty}'$. Figure 5(*a*) shows the η-dependence of ρ_p calculated in this way for $\gamma = 2$, $\kappa = 1$ and $X = 0.1$; 0.2; 0.5; 1; 1.6 and scaled by $\rho_{p\infty}'$. The maximum density near the boundary of separation is several times greater than unity.

Separation in the boundary layer occurs because of the action of the lift only. The particles at some distance from the wall have finite streamwise velocity, and the centrifugal force induces them to move outward from the boundary layer. However, this force decreases quadratically with distance from the wall and, hence, it cannot force particles near the wall to separate from it. Particle streamlines calculated with neglect of the lift are shown by dashed lines in figure 4. It is seen that dust flow does not separate in this case. Particle density distributions evaluated with neglect of the lift for the same values of γ, κ, X as in figure 5(a) are presented in figure 5(b). $\rho_p(X, 0)$ decreases with X but never become zero.

7. Discussion

The asymptotic method has been applied above to obtain the particle velocity field in the boundary layer for fixed relationships between the main dimensionless parameters. A similar approach can be used when these relations are different from (3.16). A necessary condition for it to be applicable is that the characteristic size of a body L be larger compared with λ_{St}, or, equivalently $\sigma \ll 1$. In this case, as follows from (3.7), leading order of the streamwise particle velocity equals that of the gas, so that

$$u_p = u + \sigma u_{p1} + O(\sigma^2).$$

The left-hand side of (3.7b) is small in comparison with the right-hand side. Expressing u_{p1} in terms of α one can write the leading-order equations as

$$u\frac{\partial u}{\partial x} + v_p\frac{\partial u}{\partial y} = Re_L^{-1/4}\,\sigma^{-1}\,\alpha\left(\frac{\partial u}{\partial y}\right)^{1/2}, \tag{7.1a}$$

$$(v - v_p)\,\sigma^{-1} + Re_L^{1/2}\,u^2 - Re_L^{-1/4}\,\epsilon^{-1}\,c(\alpha)\,\alpha\frac{\partial u}{\partial y} = 0. \tag{7.1b}$$

A new length scale λ_{ce} can be introduced as the length L that makes the dimensionless parameters corresponding to the normal drag and centrifugal force equal, namely

$$Re_L^{1/2} = \sigma^{-1}. \tag{7.2a}$$

or, equivalently,

$$Re_L^{1/2} = \tfrac{9}{2}Re_L\,R_V^{-2}\,\beta. \tag{7.2b}$$

Denote Re_{ce} as the Reynolds number Re_L that satisfies equation (7.2b). This gives

$$Re_{ce} = \tfrac{4}{81}\beta^{-2}\,R_V^4. \tag{7.3}$$

Defining a length scale λ_{ce} such that $Re_{ce} = V\lambda_{ce}/\nu$, one obtains from (7.3)

$$\lambda_{ce} = \tfrac{4}{81}(V/\mu)^3 a^4 \rho\rho_p^2.$$

Three length scales, for $\beta \ll 1$ and $R_V^{-1} \ll 1$, are related as follows:

$$\lambda_{St} \ll \lambda_{Sa} \ll \lambda_{ce}.$$

Since there are no derivatives of particle velocity in (7.1), again we can obtain an algebraic equation for α similar to (4.6). Eliminating v_p one can reduce (7.1) to

$$\left[\chi(\alpha)\left(\frac{\partial u}{\partial y}\right)^2 - \alpha\left(\frac{\partial u}{\partial y}\right)^{1/2}\left(\frac{L}{\lambda_{Sa}}\right)^{3/4}\right]\frac{R_V}{6\pi} + \frac{\mathrm{D}u}{\mathrm{D}t} + \frac{u^2}{r}\frac{\partial u}{\partial y}\left(\frac{L}{\lambda_{ce}}\right)^{-1/2} = 0. \tag{7.4}$$

The particle normal velocity is expressed in terms of gas velocity and α as

$$v_p = v + \frac{u^2}{r}\left(\frac{L}{\lambda_{ce}}\right)^{-1/2} + \chi(\alpha)\frac{\partial u}{\partial y}\frac{R_V}{6\pi}. \tag{7.5}$$

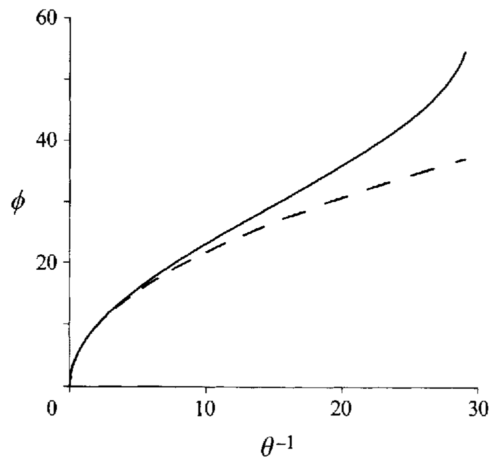

FIGURE 6. Azimuth of the separation point on the sphere as a function of the ratio of its radius and normal velocity relaxation length (solid line). Dashed line is that given by (7.10).

When the particle motion in the boundary layer over a body of size of the order of λ_{Sa} is considered, terms in (7.4) and (7.5) corresponding to centrifugal force can be estimated as

$$(\lambda_{Sa}/\lambda_{ce})^{-1/2} \sim \beta^{-1/3} \gg 1. \tag{7.6}$$

Thus in this case the streamwise acceleration of the gas in (7.4), being of the order of unity, is small in comparison with other terms. The ratio of the lift and centrifugal force depends on the relationships between β and R_V^{-1}. These forces are, from (7.4) and (7.6), of the same order when $R_V \sim \beta^{-1/3}$. In this case we have from (7.4) and (7.5) $\alpha \sim 1$ and $v_p \sim \beta^{-1/3}$. Thus the joint action of the two normal forces results in the particle normal velocity being asymptotically large compared with that of the gas. Particle density is asymptotically small in comparison with the free-stream value. However, a particle-free region again may arise owing to lift only. By analogy with §6 one can deduce from (7.4), (7.5) that the zeroth solution is unstable, and, hence, non-zero solutions for α and v_p as $y \to 0$ occur in the region where the fluid shear is greater than its critical value given by

$$\left(\frac{\partial u}{\partial y}\right)_{c0} = c(0)^{-2/3}\left(\frac{L}{\lambda_{Sa}}\right)^{1/2}. \tag{7.7}$$

The separation of dust flow occurs at the point on the wall where the fluid shear reaches a critical value. Therefore the separation point x_s can be evaluated from the equation

$$\frac{\partial u(x_s, 0)}{\partial y} = c(0)^{-2/3}\left(\frac{L}{\lambda_{Sa}}\right)^{1/2} \tag{7.8}$$

As an example, it is useful to calculate the position of separation for the boundary-layer flow of dusty gas past a sphere for arbitrary L/λ_{Sa}. Using an approximate solution (Schlichting 1968) the gas shear for this problem can be written as

$$\frac{\partial u(x,0)}{\partial y} = \tfrac{3}{2}\surd 3\phi''(0)\,(x - 0.3925x^3 + 0.0421x^5 - 0.0259x^7). \tag{7.9}$$

Here x is scaled by the radius of the sphere r'. Equation (7.8) can be easily solved numerically. The calculated value of the azimuth of the separation point introduced by $\phi = 180°x_s/\pi$ as a function of $\theta^{-1} = r'/\lambda_{Sa}$ is presented in figure 6 by solid line. When the separation occurs in the vicinity of the stagnation point, its position can be estimated by rewriting (6.2) as

$$x_s = \frac{2}{3}\frac{c(0)^{-2/3}}{\sqrt{3\phi''(0)}}\left(\frac{r'}{\lambda_{Sa}}\right)^{1/2} \approx 0,12\left(\frac{r'}{\lambda_{Sa}}\right)^{1/2}. \tag{7.10}$$

Function $\phi(\theta^{-1})$ following from (7.10) is shown by the dashed line, and is obtained for the case when the body size is small compared with the normal velocity relaxation length. It is seen however in figure 6 that this approximation agrees well with solution following from (7.9) up to $r'/\lambda_{Sa} \approx 10$.

When the radius of the sphere is reasonably large, nowhere on the surface does the fluid shear reach the critical magnitude given by (7.7). In this case dust flow does not separate. The critical regime when separation still takes place obviously occurs if the critical fluid shear is reached at the point where the shear is maximum. Performing the differentiation of (7.9) with respect to x one can easily obtain that the coordinate and azimuth of maximum shear are $x_c \approx 1.01$ and $\phi_c \approx 57, 8°$. Then we have from (7.8) that the critical radius $r_c \approx 29.2$, or in dimensional form

$$r'_c \approx 29.2\lambda_{Sa} \approx 0.0784\,(V/\mu)^3 a^4 \rho^{5/3} \rho_p^{4/3}.$$

It should be noted that the position of the separation point is dictated only by the ratio L/λ_{Sa} and does not depend on the relationships between β and R_V^{-1}. This remains valid even for $R_V \ll \beta^{-1/3}$ when over a major portion of the boundary layer the centrifugal force is dominant. In this case the lift effect on particle motion is significant at $x > x_s$ in a thin sublayer near the wall with thickness $R_V^{1/2}\beta^{-1/6}$ and again forces the dust flow to separate.

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